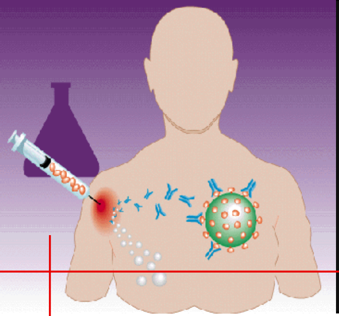

Research Week 2015

T Test, ANOVA

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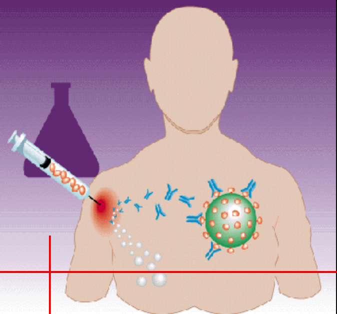
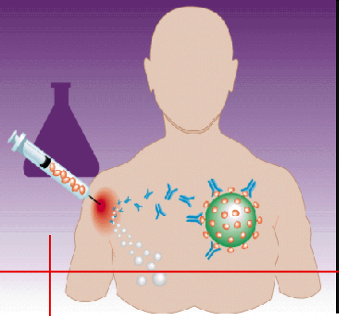
T-Test



Independent T-Test
Student's T-Test

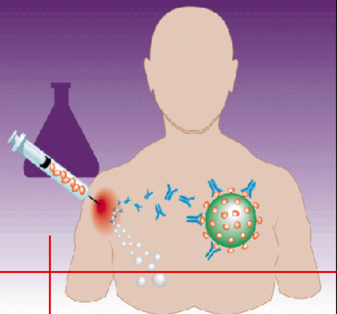
Paired T-Test

ANOVA



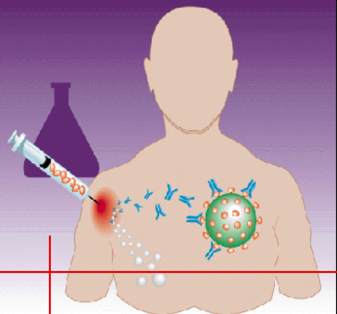
Student's T-test

William Sealy Gosset @ "Student", 1908. The Probable Error of Mean. Biometrika.



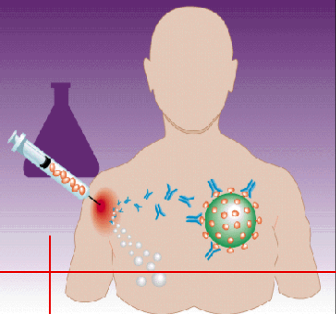
Student's T-Test

- To compare the means of two independent groups. For example; comparing the mean Hb between cases and controls. 2 variables are involved here, one quantitative (i.e. Hb) and the other a dichotomous qualitative variable (i.e. case/control).
- $$t = \frac{\bar{x}_1 - \bar{x}_2}{SEM_D}$$



Examples: Student's t-test

- Comparing the level of blood cholestrol (mg/dL) between the hypertensive and normotensive.
- Comparing the HAMD score of two groups of psychiatric patients treated with two different types of drugs (i.e. Fluoxetine & Sertraline



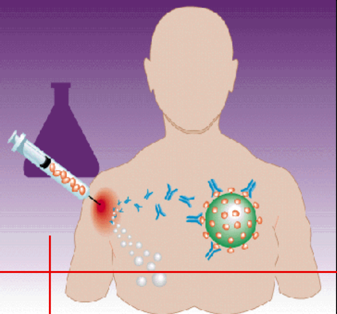
Example

Group Statistics

	DRUG	N	Mean	Std. Deviation
DHAMAWK6	F	35	4.2571	3.12808
	S	32	3.8125	4.39529

Independent Samples Test

		t-test for Equality of Means			
		t	df	Sig. (2-tailed)	Mean Difference
DHAMAWK6	Equal variances assumed	.48	65	.633	.4446



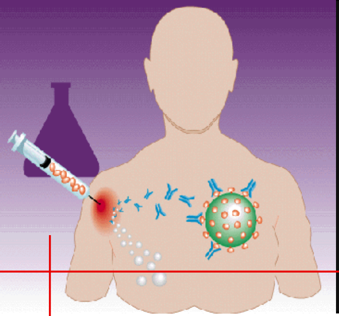
Assumptions of T test

- Observations are normally distributed in each population. (Explore)
- The population variances are equal. (Levene's Test)
- The 2 groups are independent of each other. (Design of study)

Manual Calculation

- Sample size > 30

- Small sample size, equal variance

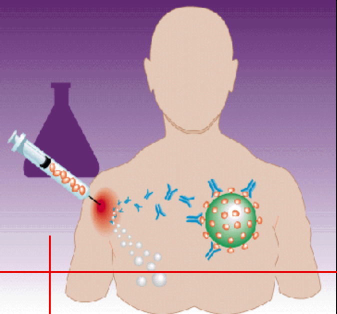


Implication

- We expect better improvement among the treated group, compared to the control group. The better the improvement, the larger is the mean difference between the two groups.
- The larger the difference, the more significant is the p value.

Significance

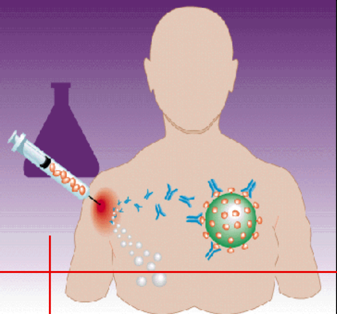
- The bigger the mean difference, the bigger is the t value, therefore the smaller is the p value, therefore more likely to be significant.
- The smaller the variance, the smaller is the p value.



Example – compare cholesterol level

- **Hypertensive :**
  Mean : 214.92
  s.d. : 39.22
  n : 64
- **Normal :**
  Mean : 182.19
  s.d. : 37.26
  n : 36

- Comparing the cholesterol level between hypertensive and normal patients.
- The difference is (214.92 – 182.19) = 32.73 mg%.
- *H0 : There is no difference of cholesterol level between hypertensive and normal patients.*
- n > 30, (64+36=100), therefore use the first formula.

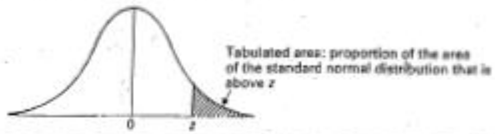


Calculation

- $t = \dfrac{(214.92 - 182.19)}{((39.22^2/64)+(37.26^2/36))^{0.5}}$
- $t = 4.137$
- $df = n_1+n_2-2 = 64+36-2 = 98$
- Refer to t table; with $t = 4.137$, $p < 0.001$

Table A1 Areas in tail of the standard normal distribution.

Adapted from Table 3 of White et al. (1979) with permission of the authors and publishers.



Second decimal place of z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

If df≥100, can refer Table A1. We don't have 4.137 so we use 3.99 instead. If t = 3.99, then p=0.00003x2=0.00006

4.137>3.99

If t=3.99, then p=0.00006

Therefore if t=4.137, p<0.00006.

3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

Table A3 Percentage points of the *t* distribution.

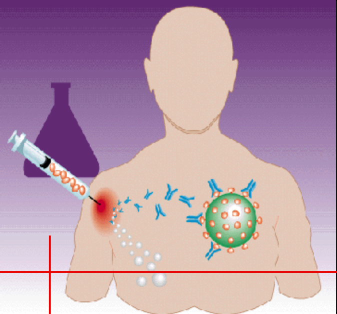
Adapted from Table 7 of White *et al.* (1979) with permission of authors and publishers.

	One-sided *P* value								
	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
	Two-sided *P* value								
d.f.	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	318.31	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	22.33	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	10.21	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	7.17	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	5.89	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.21	5.96
7	0.71	1.42	1.90	2.36	3.00	3.50	4.03	4.78	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	4.50	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.30	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.14	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.02	4.44
12	0.70	1.36	1.78	2.18	2.68	3.06	3.43	3.93	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	3.85	4.22
14	0.69	1.34	1.76	2.14	2.62	2.98	3.33	3.79	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	3.73	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	3.69	4.02
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.65	3.96
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.61	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.58	3.88
20	0.69	1.32	1.72	2.09	2.53	2.84	3.15	3.55	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.53	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.50	3.79
23	0.68	1.32	1.71	2.07	2.50	2.81	3.10	3.48	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.47	3.74
25	0.68	1.32	1.71	2.06	2.48	2.79	3.08	3.45	3.72
26	0.68	1.32	1.71	2.06	2.48	2.78	3.07	3.44	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.42	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.41	3.67
29	0.68	1.31	1.70	2.04	2.46	2.76	3.04	3.40	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.38	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.31	3.55
60	0.68	1.30	1.67	2.00	2.39	2.66	2.92	3.23	3.46
120	0.68	1.29	1.66	1.98	2.36	2.62	2.86	3.16	3.37
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.09	3.29

Or can refer to Table A3.
We don't have df=98,
so we use df=60 instead.
t = 4.137 > 3.46

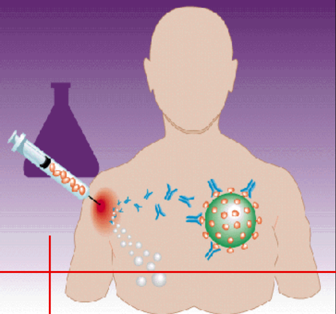
If t=3.46, p=0.001
Therefore if t=4.137, p<0.001.

	Two-sided *P* value								
d.f.	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.38	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.31	3.55
60	0.68	1.30	1.67	2.00	2.39	2.66	2.92	3.23	3.46
120	0.68	1.29	1.66	1.98	2.36	2.62	2.86	3.16	3.37
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.09	3.29



Conclusion

- Therefore $p < 0.05$, null hypothesis rejected.
- *There is a significant difference of cholesterol level between hypertensive and normal patients.*
- Hypertensive patients have a **significantly** higher cholesterol level (215±39) compared to normotensive (182±37) patients.

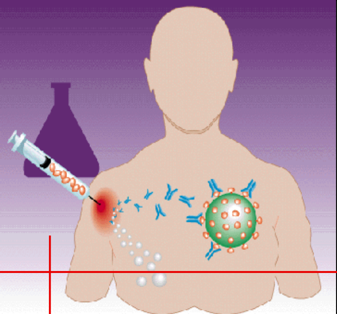


Exercise (try it)

- Comparing the mini test 1 (2012) results between UKM and ACMS students.
- The difference is 11.255
- *H0 : There is no difference of marks between UKM and ACMS students.*
- n > 30, therefore use the first formula.

Group Statistics

	group	N	Mean	Std. Deviation
minitest1	UKM	196	58.83	14.129
	AUCMS	70	47.57	14.289



Exercise (answer)

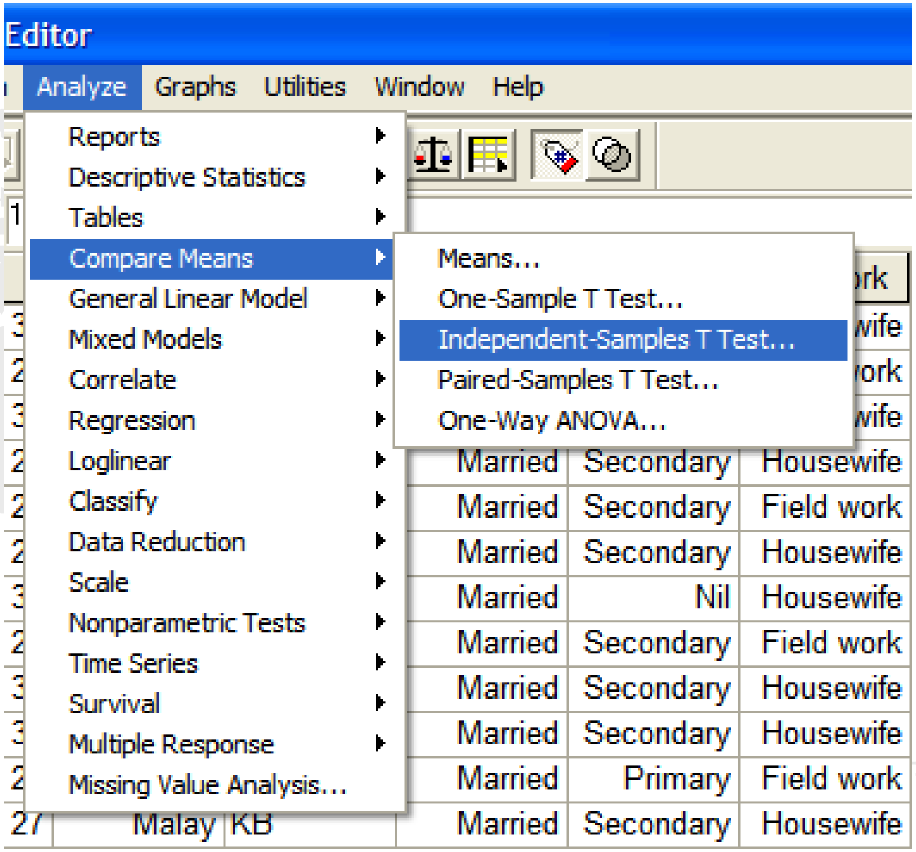
Independent Samples Test

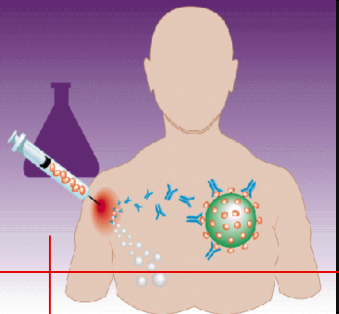
		t-test for Equality of Means			
		t	df	Sig. (2-tailed)	Mean Difference
minitest1	Equal variances assumed	5.704	264	0.00000003	11.255

- *Null hypothesis rejected*
- *There is a difference of marks between UKM and ACMS students. UKM marks higher than AUCMS*

T-Test In SPSS

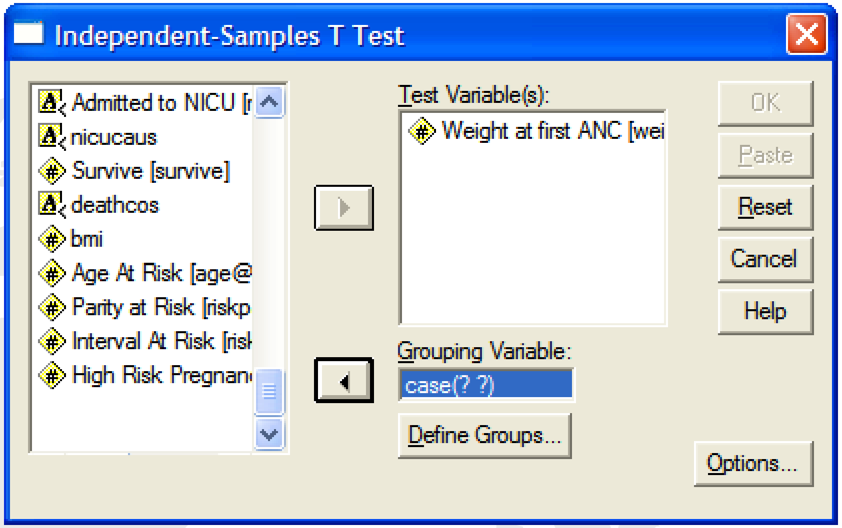
- For this exercise, we will be using the data from the CD, under Chapter 7, sga-bab7.sav
- This data came from a case-control study on factors affecting SGA in Kelantan.
- Open the data & select ->Analyse
    >Compare Means
        >Ind-Samp T Test…

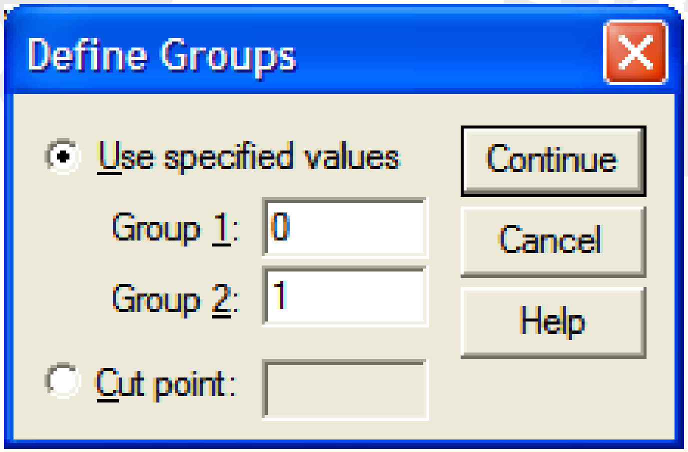


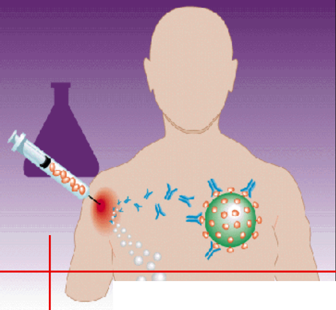


T-Test in SPSS

- We want to see whether there is any association between the mothers' weight and SGA. So select the risk factor (weight2) into 'Test Variable' & the outcome (SGA) into 'Grouping Variable'.
- Now click on the 'Define Groups' button. Enter
  - 0 (Control) for Group 1 and
  - 1 (Case) for Group 2.
- Click the 'Continue' button & then click the 'OK' button.





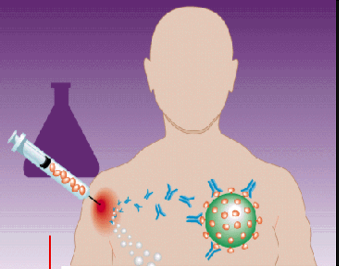


T-Test Results

Group Statistics

	SGA	N	Mean	Std. Deviation	Std. Error Mean
Weight at first ANC	Normal	108	58.666	11.2302	1.0806
	SGA	109	51.037	9.3574	.8963

- Compare the mean±sd of both groups.
  - Normal 58.7±11.2 kg
  - SGA 51.0± 9.4 kg
- Apparently there is a difference of weight between the two groups.

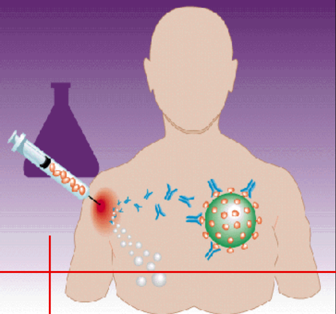


Results & Homogeneity of Variances

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Weight at first ANC	Equal variances assumed	1.862	.174	5.439	215	.000	7.629	1.4028	4.8641	10.3940
	Equal variances not assumed			5.434	207.543	.000	7.629	1.4039	4.8612	10.3969

- Look at the p value of Levene's Test. If p is not significant then equal variances is assumed (use top row).
- If it is significant then equal variances is not assumed (use bottom row).
- So the t value here is 5.439 and $p < 0.0005$. The difference is significant. Therefore there is an association between the mothers weight and SGA.

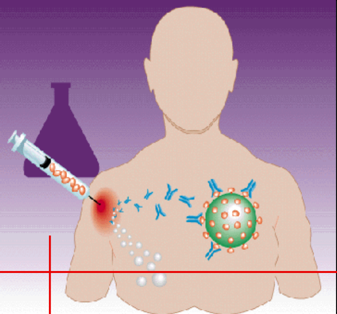


How to present the result?

Group	N	Mean	test	p
Normal	108	58.7±11.2 kg	T test t = 5.439	<0.0005
SGA	109	51.0± 9.4		

Paired t-test

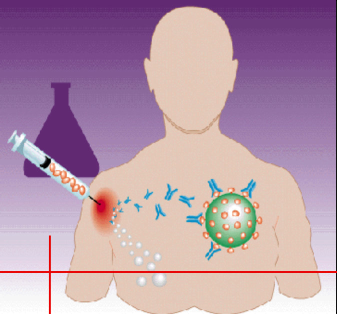
“Repeated measurement on the same individual”



Paired T-Test

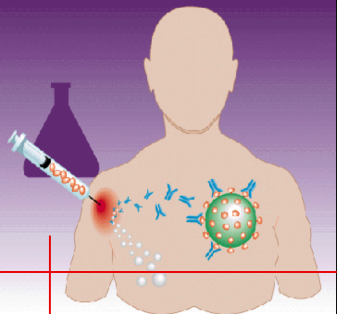
- "Repeated measurement on the same individual"
- $$t = \frac{\bar{D} - 0}{SEM_D}$$

Formula



Examples of paired t-test

- Comparing the HAMD score between week 0 and week 6 of treatment with Sertraline for a group of psychiatric patients.
- Comparing the haemoglobin level amongst anaemic pregnant women after 6 weeks of treatment with haematinics.



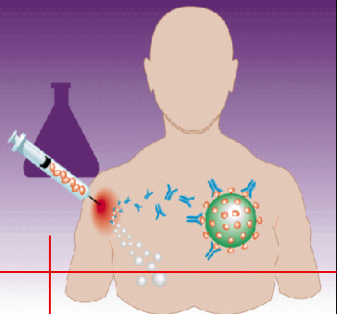
Example

Paired Samples Statistics

		Mean	N	Std. Deviation
Pair 1	DHAMAWK0	13.9688	32	6.48315
	DHAMAWK6	3.8125	32	4.39529

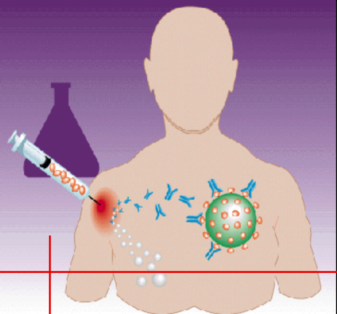
Paired Samples Test

		Paired Differences		t	df	Sig. (2-tailed)
		Mean	Std. Deviation			
Pair 1	DHAMAWK0 - DHAMAWK6	10.1563	6.75903	8.500	31	.000



Manual Calculation

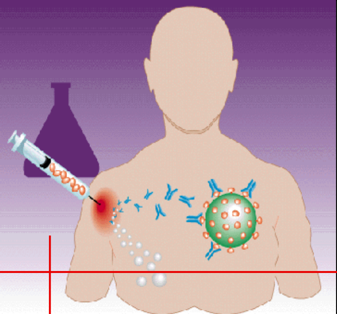
- *The measurement of the systolic and diastolic blood pressures was done two consecutive times with an interval of 10 minutes. You want to determine whether there was any difference between those two measurements.*
- *H0:There is no difference of the systolic blood pressure during the first (time 0) and second measurement (time 10 minutes).*



Calculation

- Calculate the difference between first & second measurement and square it. Total up the difference and the square.

Nores	BPS1	BPS2	d	d^2
232	164	163	1.00	1.00
233	164	155	9.00	81.00
236	156	158	-2.00	4.00
237	147	131	16.00	256.00
239	186	178	8.00	64.00
241	170	160	10.00	100.00



Calculation

- $\sum d = 112$ $\sum d^2 = 1842$ $n = 36$
- Mean d = 112/36 = 3.11
- sd = $((1842-112^2/36)/35)^{0.5}$

  sd = 6.53
- t = 3.11/(6.53/6)

  t = 2.858
- df = $n_p - 1 = 36 - 1 = 35$.
- Refer to t table;

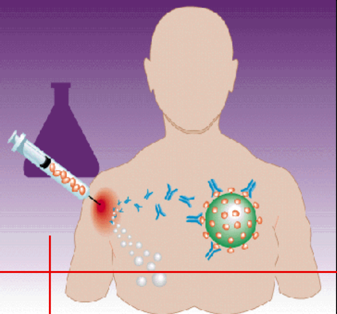
Table A3 Percentage points of the *t* distribution.

Adapted from Table 7 of White *et al.* (1979) with permission of authors and publishers.

	One-sided *P* value								
	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
	Two-sided *P* value								
d.f.	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	318.31	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	22.33	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	10.21	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	7.17	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	5.89	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.21	5.96
7	0.71	1.42	1.90	2.36	3.00	3.50	4.03	4.78	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	4.50	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.30	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.14	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.02	4.44
12	0.70	1.36	1.78	2.18	2.68	3.06	3.43	3.93	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	3.85	4.22
14	0.69	1.34	1.76	2.14	2.62	2.98	3.33	3.79	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	3.73	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	3.69	4.02
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.65	3.96
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.61	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.58	3.88
20	0.69	1.32	1.72	2.09	2.53	2.84	3.15	3.55	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.53	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.50	3.79
23	0.68	1.32	1.71	2.07	2.50	2.81	3.10	3.48	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.47	3.74
25	0.68	1.32	1.71	2.06	2.48	2.79	3.08	3.45	3.72
26	0.68	1.32	1.71	2.06	2.48	2.78	3.07	3.44	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.42	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.41	3.67
29	0.68	1.31	1.70	2.04	2.46	2.76	3.04	3.40	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.38	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.31	3.55
60	0.68	1.30	1.67	2.00	2.39	2.66	2.92	3.23	3.46
120	0.68	1.29	1.66	1.98	2.36	2.62	2.86	3.16	3.37
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.09	3.29

Refer to Table A3.
We don't have df=35,
so we use df=30 instead.
t = 2.858, larger than 2.75
(p=0.01) but smaller than 3.03
(p=0.005). 3.03>t>2.75
Therefore if t=2.858,
$0.005<p<0.01$.

	Two-sided *P* value						
d.f.	0.5	0.2	0.1	0.05	0.02	0.01	0.005
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97
60	0.68	1.30	1.67	2.00	2.39	2.66	2.92
120	0.68	1.29	1.66	1.98	2.36	2.62	2.86
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81



Conclusion

with $t = 2.858$, $0.005 < p < 0.01$

Therefore $p < 0.01$.

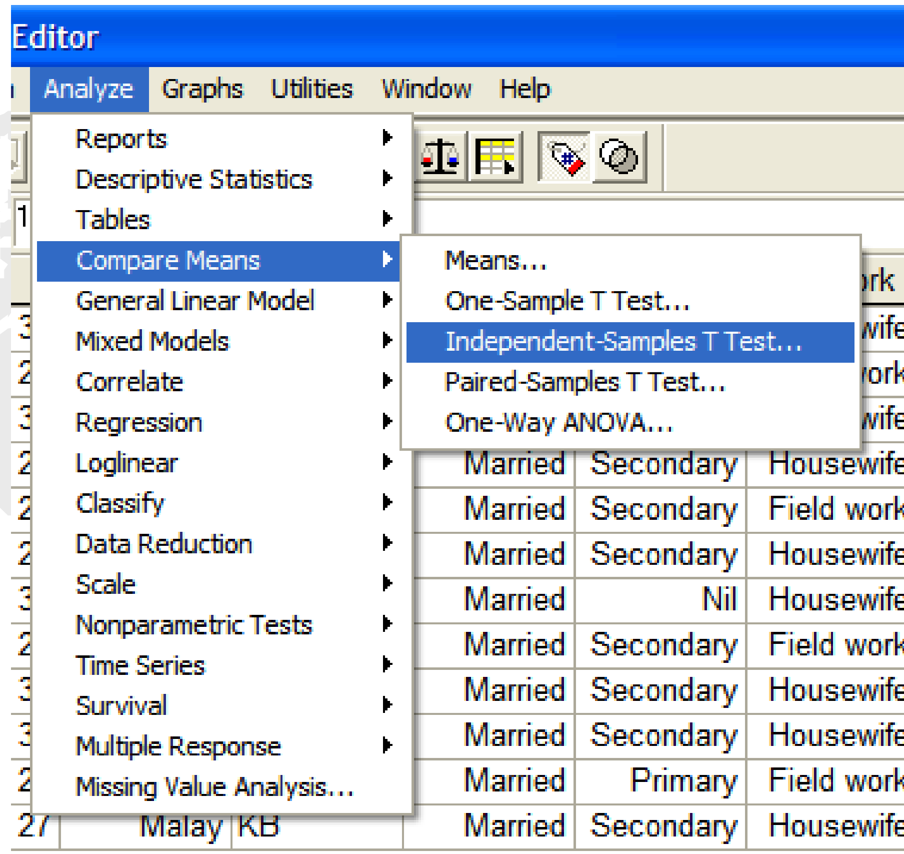
Therefore $p < 0.05$, null hypothesis rejected.

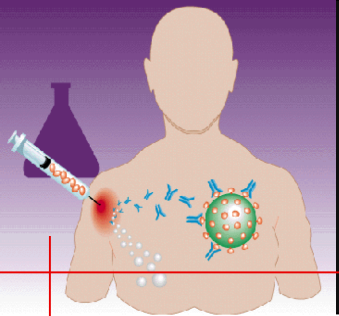
Conclusion: *There is a significant difference of the systolic blood pressure between the first and second measurement. The mean average of first reading is significantly higher (by 3.11 ±6.53) compared to the second reading.*

Paired T-Test In SPSS

- For this exercise, we will be using the data from the CD, under Chapter 7, sgapair.sav
- This data came from a controlled trial on haematinic effect on Hb.
- Open the data & select ->Analyse
  >Compare Means
    >Paired-Samples T

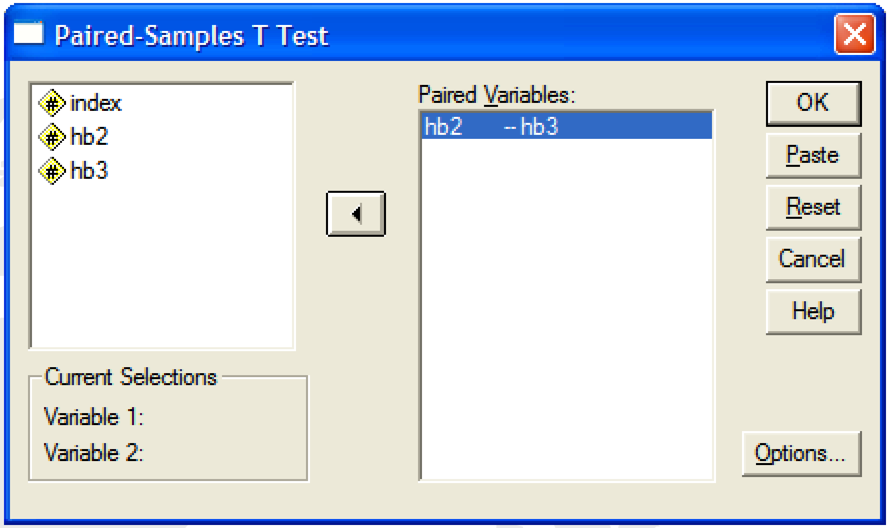
  Test…





Paired T-Test In SPSS

- We want to see whether there is any association between the prescription on haematinic to anaemic pregnant mothers and Hb.
- We are comparing the Hb before & after treatment. So pair the two measurements (Hb2 & Hb3) together.
- Click the ‘OK’ button.

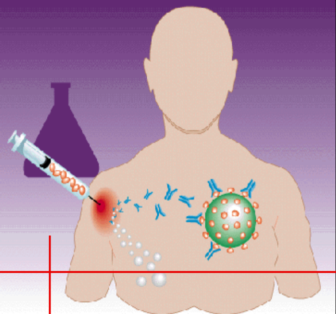


Paired T-Test Results

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	HB2	10.247	70	.3566	.0426
	HB3	10.594	70	.9706	.1160

- This shows the mean & standard deviation of the two groups.

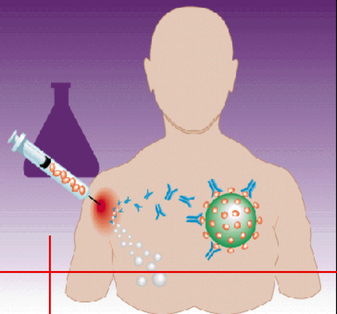


Paired T-Test Results

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	95% Confidence Interval of the Difference: Upper			
Pair 1	HB2 - HB3	-.347	.9623	.1150	-.577	-.118	-3.018	69	.004

- This shows the mean difference of Hb before & after treatment is only 0.347 g%.
- Yet the t=3.018 & p=0.004 show the difference is statistically significant.



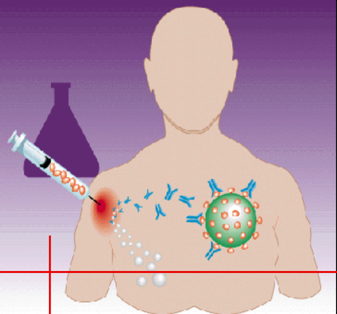
How to present the result?

Group	N	Mean D (Diff.)	Test	p
Before treatment (HB2) vs After treatment (HB3)	70	0.35 ± 0.96	Paired T-test t = 3.018	0.004

ANOVA

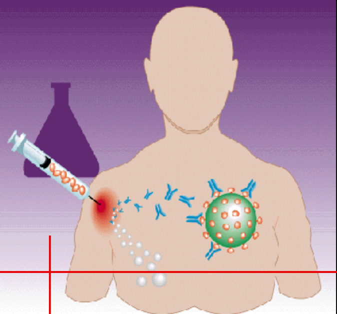
ANOVA – Analysis of Variance

- Extension of independent-samples t test
- Compares the means of groups of independent observations
  - Don't be fooled by the name. ANOVA does not compare variances.
- Can compare more than two groups



One-Way ANOVA F-Test

- Tests the equality of 2 or more population means
- Variables
  - One nominal scaled independent variable
    - 2 or more treatment levels or classifications (i.e. Race; Malay, Chinese, Indian & Others)
  - One interval or ratio scaled dependent variable (i.e. weight, height, age)
- Used to analyse completely randomized experimental designs

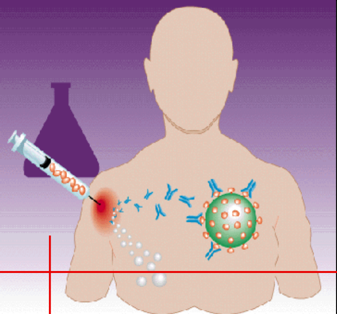


Examples

- Comparing the blood cholesterol levels between the bus drivers, bus conductors and taxi drivers.
- Comparing the mean systolic pressure between Malays, Chinese, Indian & Others.

One-Way ANOVA F-Test Assumptions

- Randomness & independence of errors
  - Independent random samples are drawn
- Normality
  - Populations are normally distributed
- Homogeneity of variance
  - Populations have equal variances



Example

Descriptives

Birth weight

	N	Mean	Std. Deviation	Minimum	Maximum
Housewife	151	2.7801	.52623	1.90	4.72
Office work	23	2.7643	.60319	1.60	3.96
Field work	44	2.8430	.55001	1.90	3.79
Total	218	2.7911	.53754	1.60	4.72

ANOVA

Birth weight

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.153	2	.077	.263	.769
Within Groups	62.550	215	.291		
Total	62.703	217			

Manual Calculation

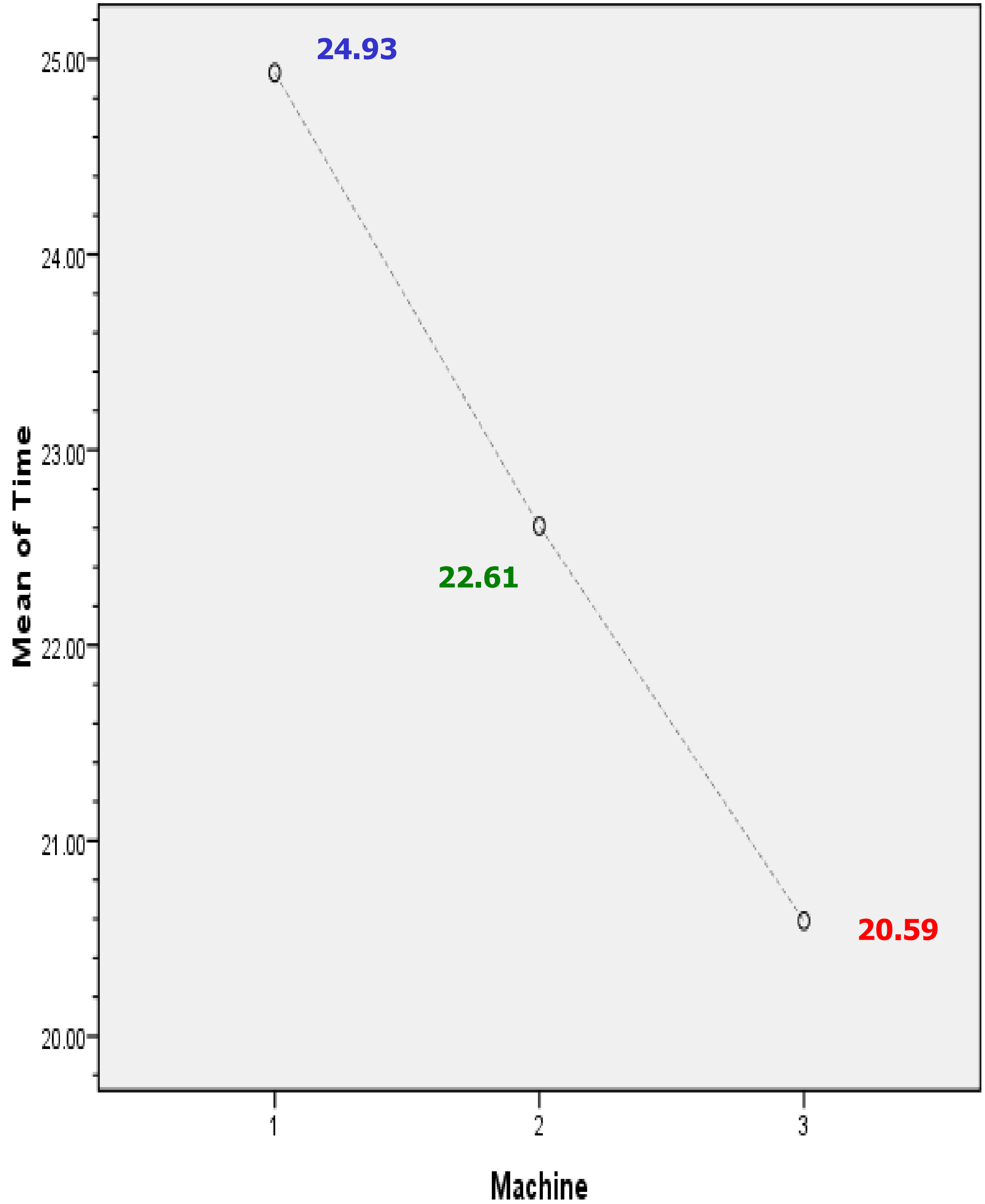
ANOVA

Example: Time To Complete Analysis

45 samples were analysed using 3 different blood analyser (Mach1, Mach2 & Mach3).

15 samples were placed into each analyser.

Time in seconds was measured for each sample analysis.

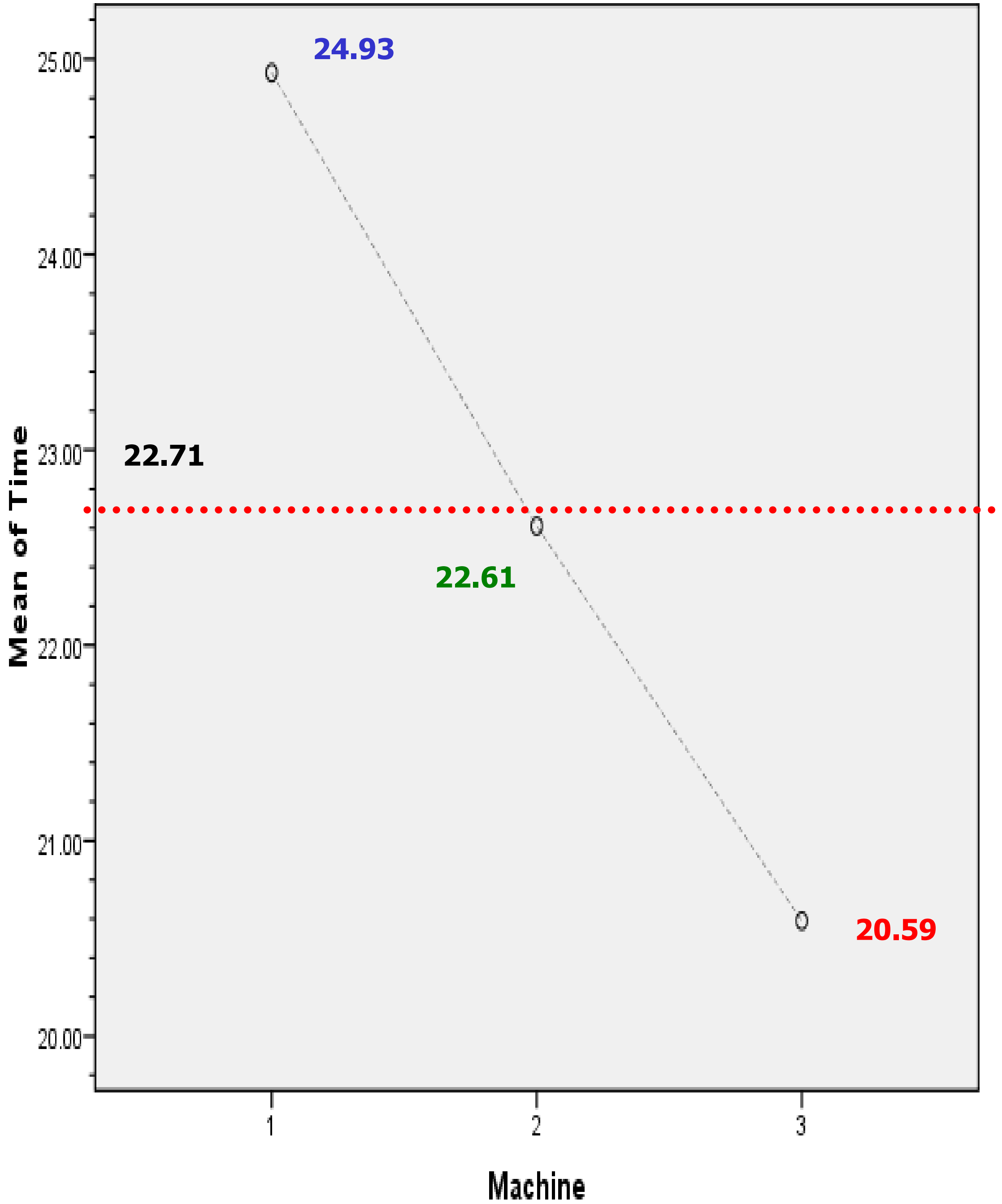


Example: Time To Complete Analysis

The overall mean of the entire sample was 22.71 seconds.

This is called the “grand” mean, and is often denoted by $\bar{\bar{X}}$.

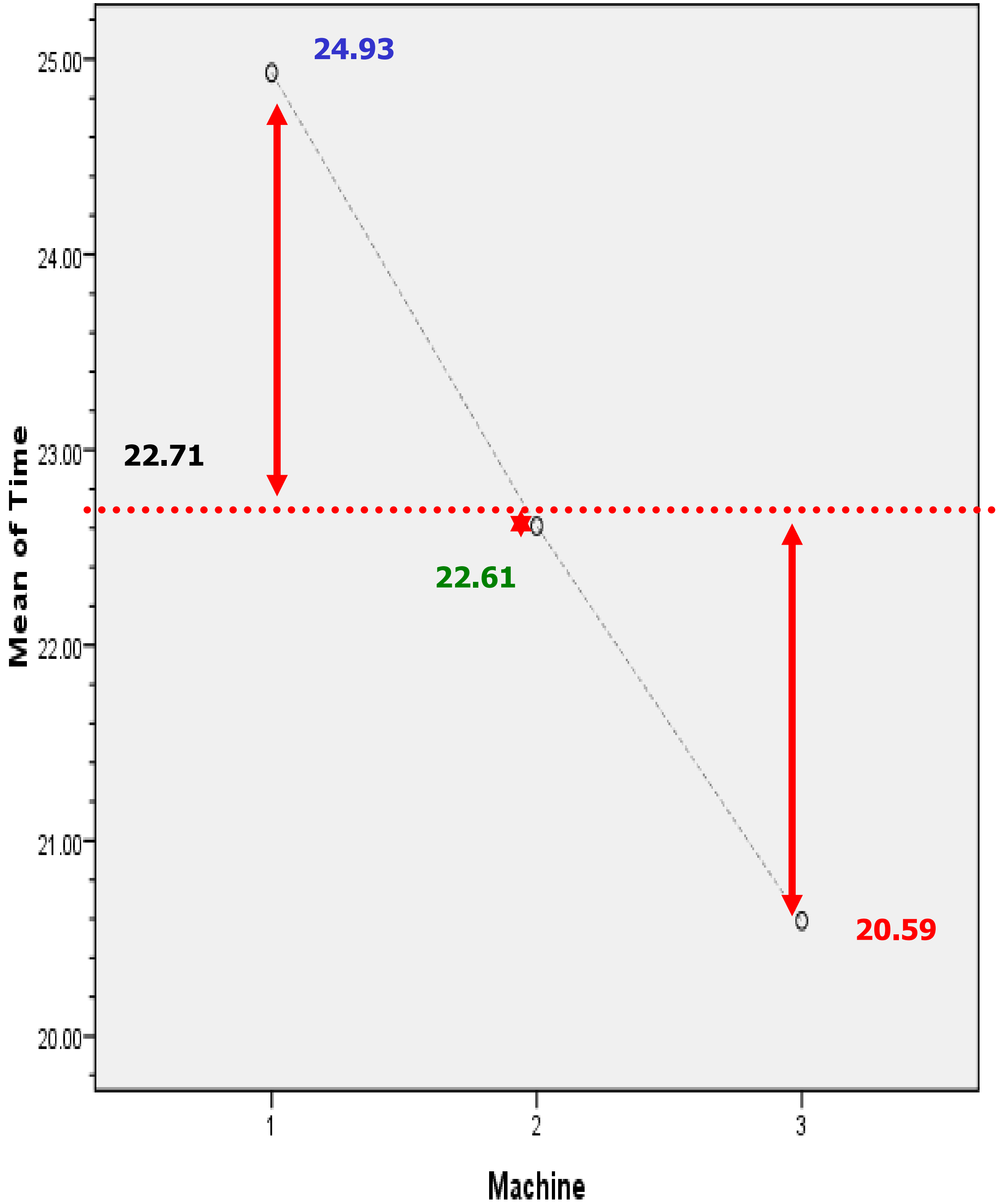
If H_0 were true then we’d expect the group means to be close to the grand mean.



Example: Time To Complete Analysis

The ANOVA test is based on the combined distances from $\bar{\bar{X}}$.

If the combined distances are large, that indicates we should reject H_0.



The Anova Statistic

To combine the differences from the grand mean we

- Square the differences
- Multiply by the numbers of observations in the groups
- Sum over the groups

$$\mathrm{SSB} = 15\left(\overline{X}_{Mach1} - \overline{\overline{X}}\right)^2 + 15\left(\overline{X}_{Mach2} - \overline{\overline{X}}\right)^2 + 15\left(\overline{X}_{Mach3} - \overline{\overline{X}}\right)^2$$

where the $\overline{X}_{*}$ are the group means.

"*SSB*" = ***S***um of ***S***quares ***B***etween groups

The Anova Statistic

To combine the differences from the grand mean we

- Square the differences
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where the $\overline{X}_*$ are the group means.

"*SSB*" = ***S***um of ***S***quares ***B***etween groups

Note: This looks a bit like a variance.

Sum of Squares Between

$$SSB = 15\left(\overline{X}_{Mach1} - \overline{\overline{X}}\right)^2 + 15\left(\overline{X}_{Mach2} - \overline{\overline{X}}\right)^2 + 15\left(\overline{X}_{Mach3} - \overline{\overline{X}}\right)^2$$

- Grand Mean = 22.71
- Mean Mach1 = 24.93; $(24.93-22.71)^2$=4.9284
- Mean Mach2 = 22.61; $(22.61-22.71)^2$=0.01
- Mean Mach3 = 20.59; $(20.59-22.71)^2$=4.4944
- SSB = (15*4.9284)+(15*0.01)+(15*4.4944)
- SSB = 141.492

How big is big?

- For the Time to Complete, *SSB* = 141.492
- Is that big enough to reject H_0?
- As with the *t* test, we compare the statistic to the variability of the individual observations.
- In ANOVA the variability is estimated by the Mean Square Error, or MSE

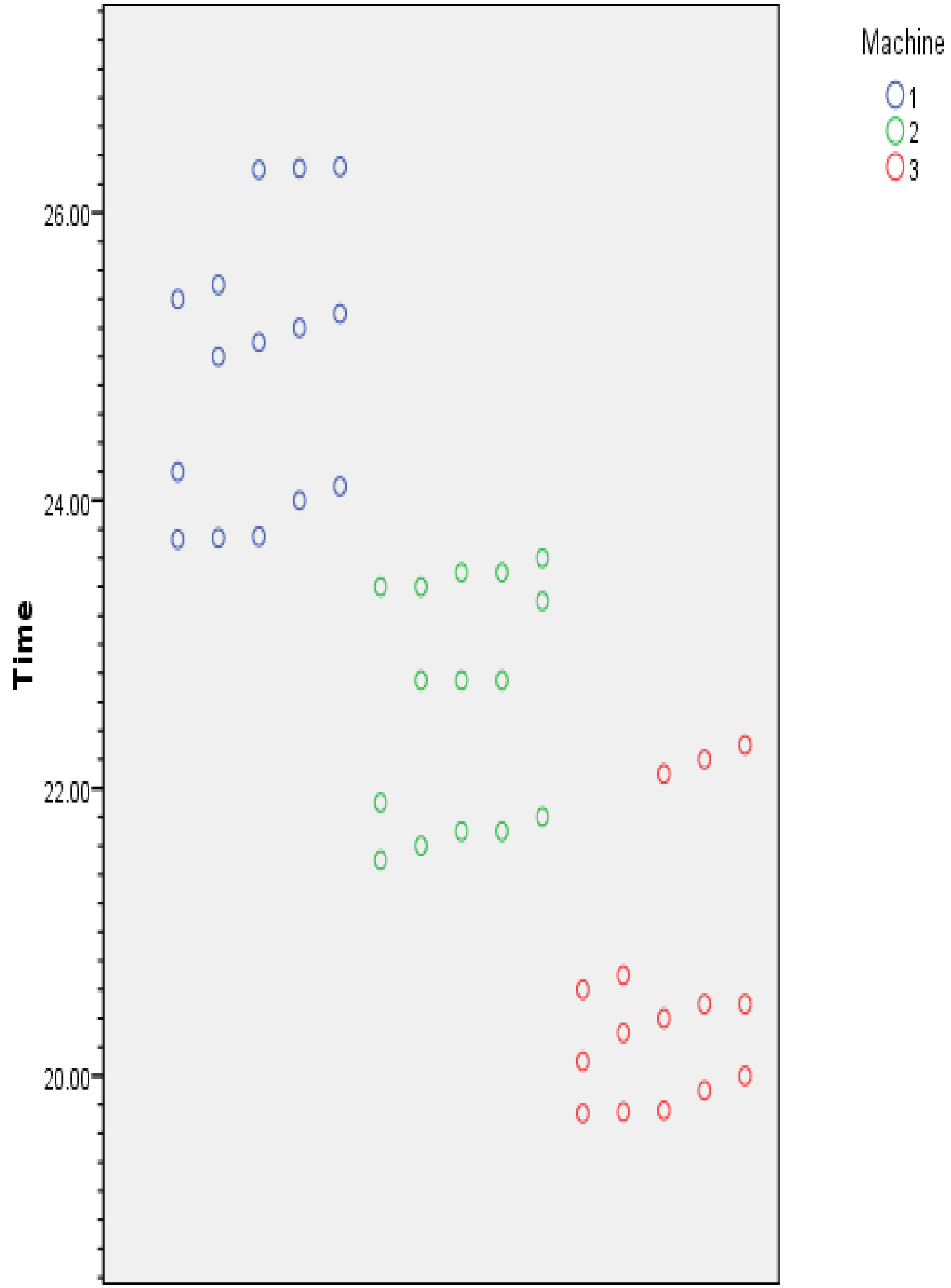
MSE

Mean Square Error

The Mean Square Error is a measure of the variability after the group effects have been taken into account.

$$MSE = \frac{1}{N-K} \sum_{j} \sum_{i} \left(x_{ij} - \overline{X}_j\right)^2$$

where x_{ij} is the i^{th} observation in the j^{th} group.



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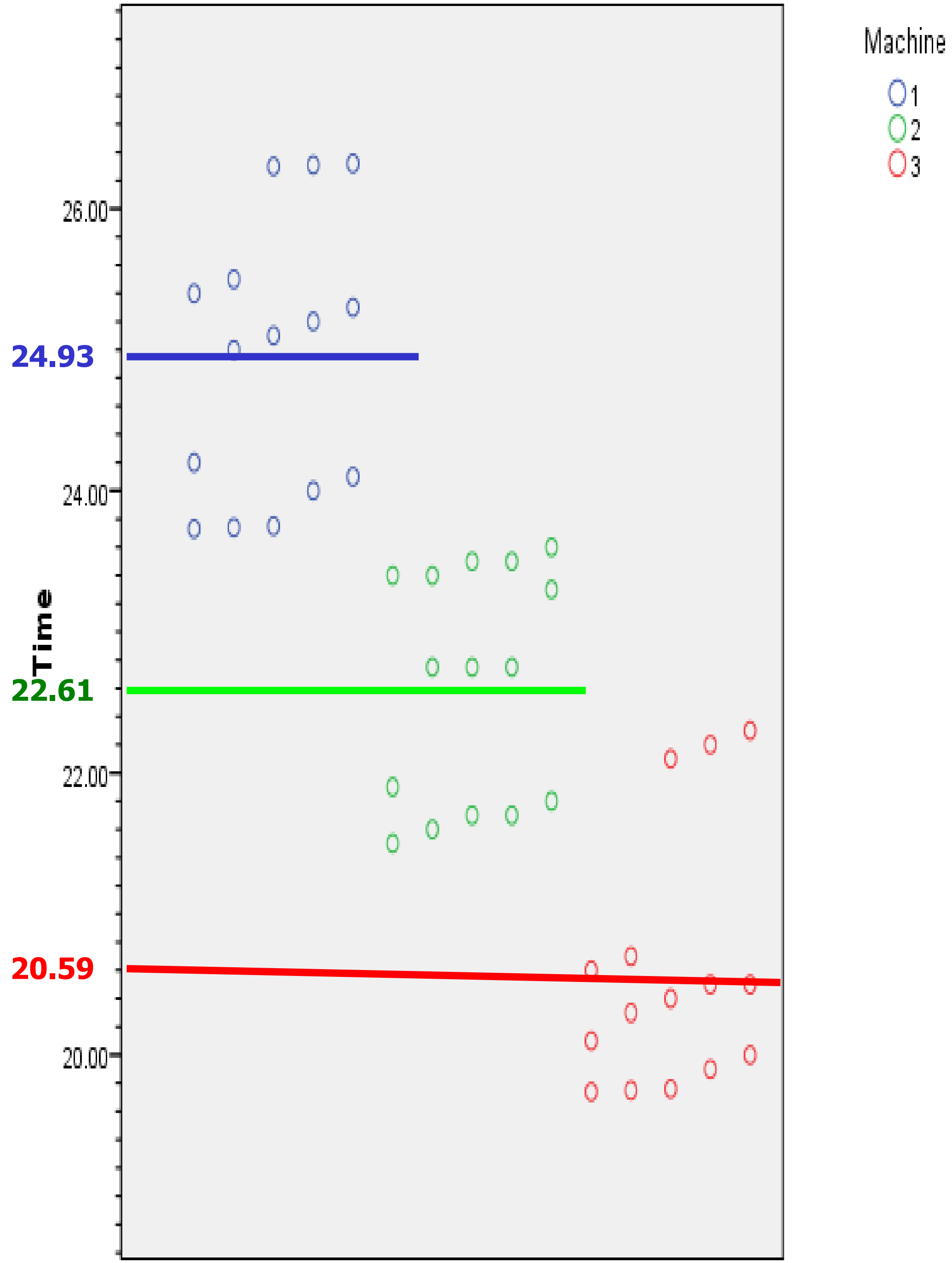

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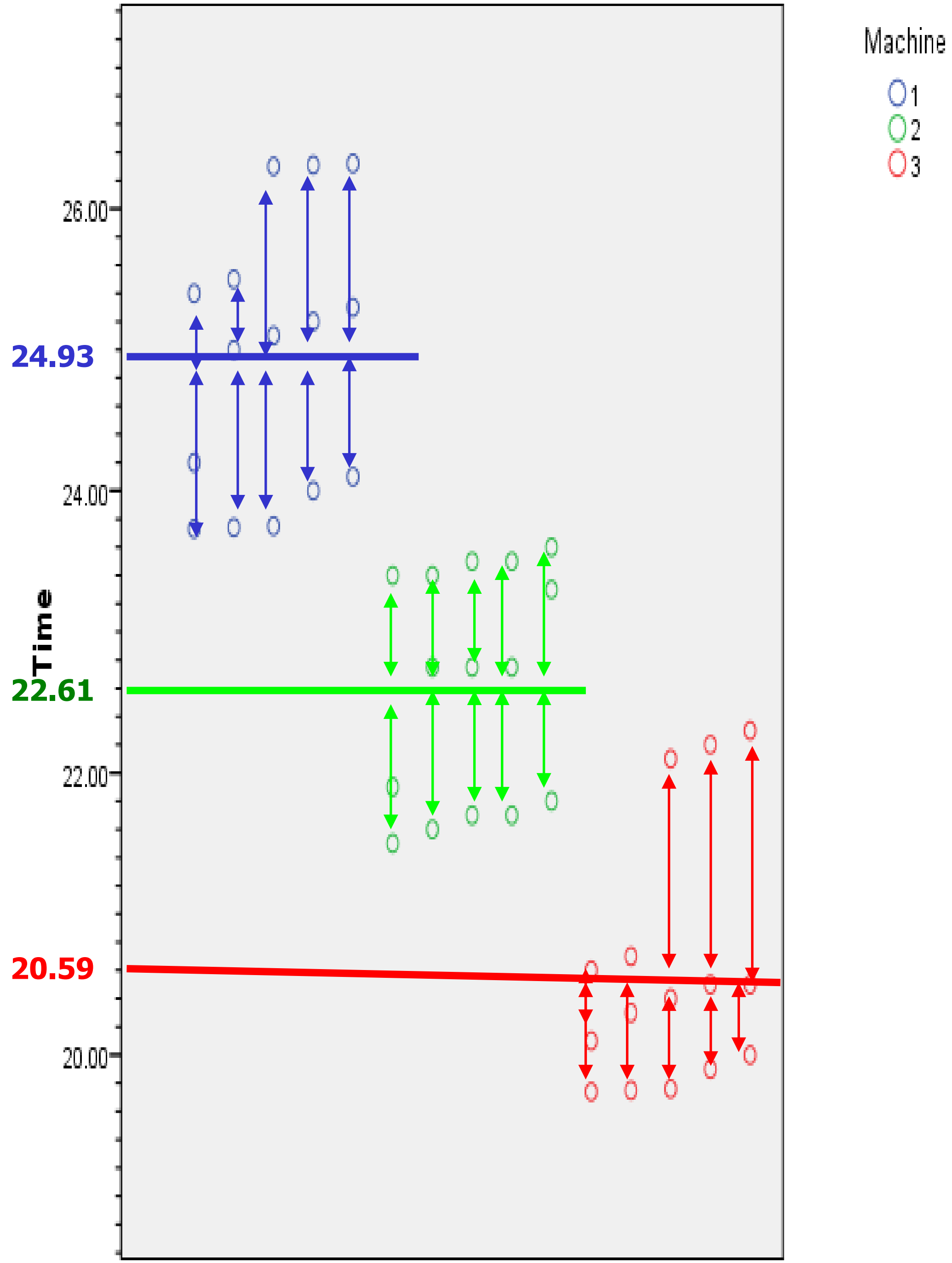
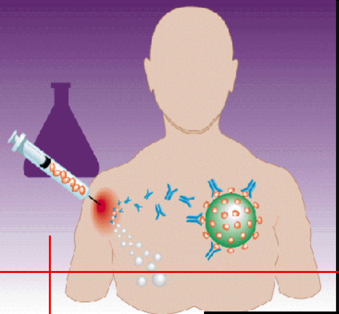
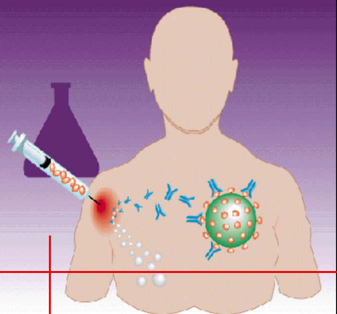




$$MSE = \frac{1}{N-K}\sum_{j}\sum_{i}\left(x_{ij} - \bar{x}_{j}\right)^2$$

Mach1	(x-mean)^2	Mach2	(x-mean)^2	Mach3	(x-mean)^2
23.73	1.4400	21.5	1.2321	19.74	0.7225
23.74	1.4161	21.6	1.0201	19.75	0.7056
23.75	1.3924	21.7	0.8281	19.76	0.6889
24.00	0.8649	21.7	0.8281	19.9	0.4761
24.10	0.6889	21.8	0.6561	20	0.3481
24.20	0.5329	21.9	0.5041	20.1	0.2401
25.00	0.0049	22.75	0.0196	20.3	0.0841
25.10	0.0289	22.75	0.0196	20.4	0.0361
25.20	0.0729	22.75	0.0196	20.5	0.0081
25.30	0.1369	23.3	0.4761	20.5	0.0081
25.40	0.2209	23.4	0.6241	20.6	0.0001
25.50	0.3249	23.4	0.6241	20.7	0.0121
26.30	1.8769	23.5	0.7921	22.1	2.2801
26.31	1.9044	23.5	0.7921	22.2	2.5921
26.32	1.9321	23.6	0.9801	22.3	2.9241
SUM	12.8380		9.4160		11.1262



$$MSE = \frac{1}{N-K}\sum_{j}\sum_{i}\left(x_{ij} - \bar{X}_{j}\right)^2$$

- Note that the variation of the means (141.492) seems quite large (more likely to be significant???) compared to the variance of observations within groups (12.8380+9.4160+11.1262=33.3802).
- MSE = 33.3802/(45-3) = 0.7948

Notes on *MSE*

- If there are only two groups, the *MSE* is equal to the pooled estimate of variance used in the equal-variance *t* test.
- ANOVA assumes that all the group variances are equal.
- Other options should be considered if group variances differ by a factor of 2 or more.
- (12.8380 ~ 9.4160 ~ 11.1262)

ANOVA F Test

- The ANOVA F test is based on the *F* statistic

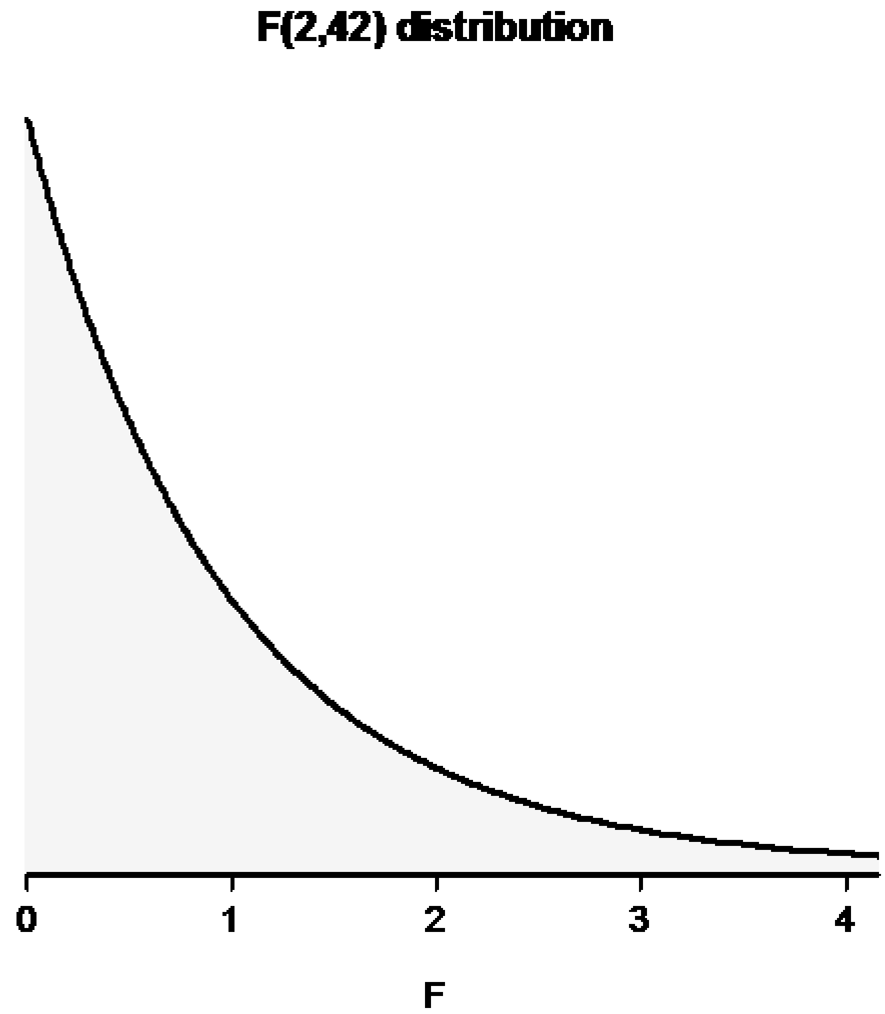
$$F = \frac{SSB/(K-1)}{MSE}$$

where *K* is the number of groups.

- Under H_0 the *F* statistic has an “F” distribution, with K-1 and N-K degrees of freedom (N is the total number of observations)

Time to Analyse: F test p-value

To get a p-value we compare our *F* statistic to an F(2, 42) distribution.



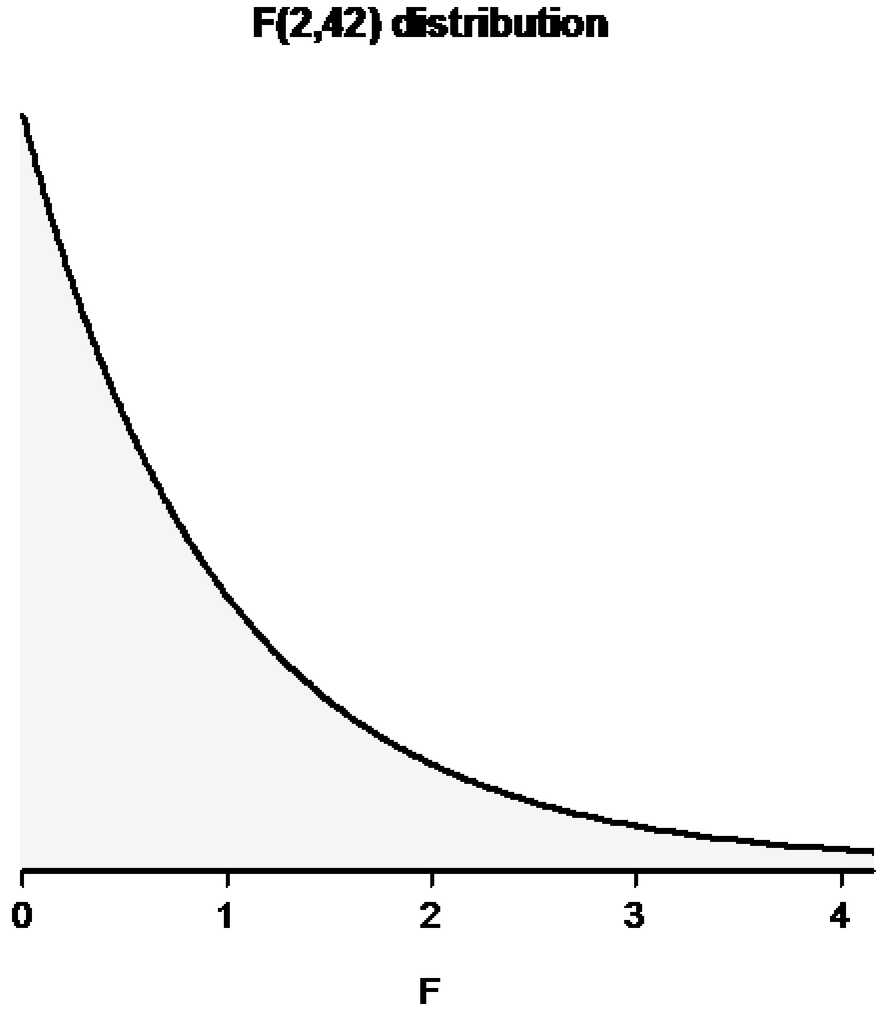
Time to Analyse: F test p-value

To get a p-value we compare our *F* statistic to an F(2, 42) distribution.

In our example

$$F = \frac{141.492/2}{33.3802/42} = 89.015$$

We cannot draw the line since the F value is so large, therefore the p value is so small!!!!!!



Appendix F: Critical Values of the F Distribution

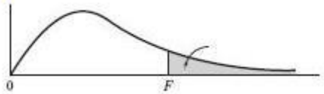


Table 1 α = 0.05

Degrees of Freedom for Numerator

Degrees of Freedom for Denominator	1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	
1	161.4	199.5	215.8	224.8	230.0	233.8	236.5	238.6	240.1	242.1	245.2	248.4	248.9	250.5	250.8	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.44	19.46	19.47	19.48	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.63	8.62	8.59	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.77	5.75	5.72	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.52	4.50	4.46	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.83	3.81	3.77	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.40	3.38	3.34	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.11	3.08	3.04	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.89	2.86	2.83	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.73	2.70	2.66	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.60	2.57	2.53	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.50	2.47	2.43	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.41	2.38	2.34	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.34	2.31	2.27	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.28	2.25	2.20	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.23	2.19	2.15	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.18	2.15	2.10	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.14	2.11	2.06	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.11	2.07	2.03	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.07	2.04	1.99	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	2.02	1.98	1.94	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.97	1.94	1.89	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.94	1.90	1.85	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.04	1.96	1.91	1.87	1.82	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.88	1.84	1.79	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.78	1.74	1.69	
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.87	1.78	1.73	1.69	1.63	[illegible]
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.84	1.75	1.69	1.65	1.59	1.56
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.75	1.66	1.60	1.55	1.50	1.46
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.72	1.62	1.56	1.52	1.46	1.41
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.69	1.59	1.53	1.48	1.42	1.38
1000	3.85	3.01	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.68	1.58	1.52	1.47	1.41	1.36

Refer to F Dist. Table (α=0.01). We don't have df=2;42, so we use df=2;40 instead. F = 89.015, larger than 5.18 (p=0.01)

Therefore if F=89.015, p<0.01.

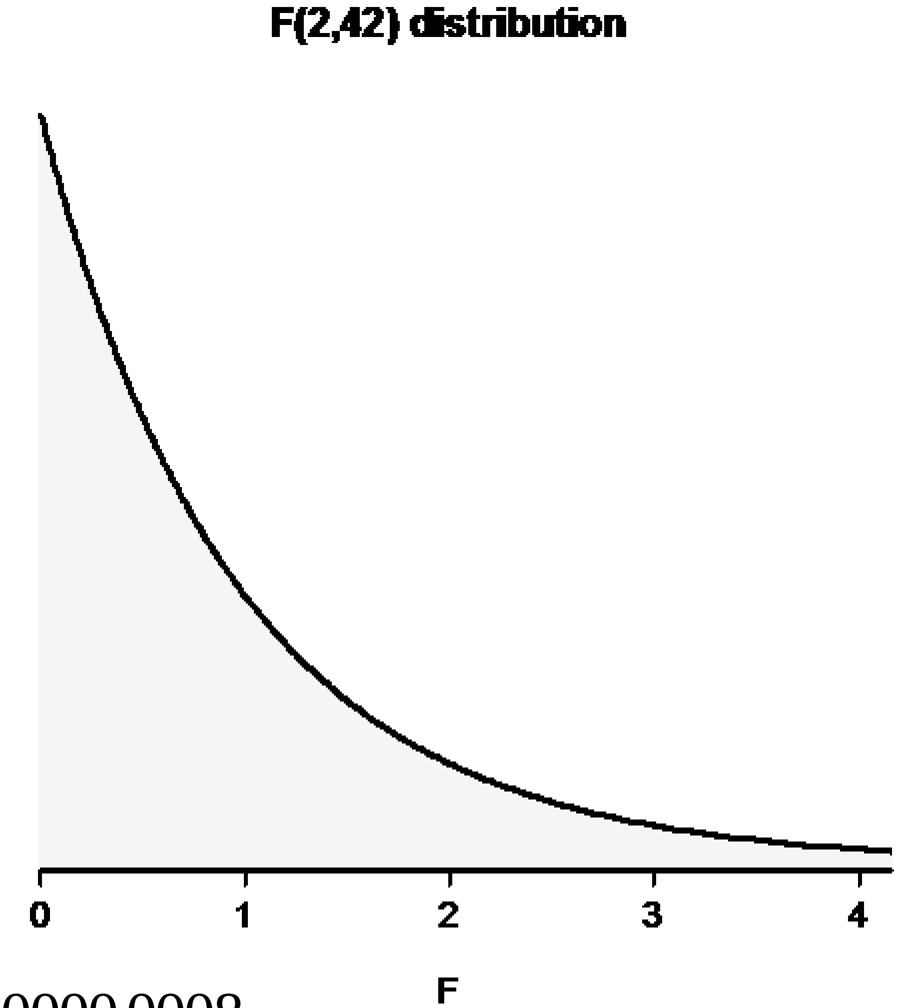
Why use df=2;42? We have K=3 groups so K-1 = 2 We have N=45 samples therefore N-K = 42.

α = 0.01

	1	2	3	4	5
30	7.56	5.39	4.51	4.02	3.70
40	7.31	5.18	4.31	3.83	3.51
50	7.17	5.06	4.20	3.72	3.41
60	7.08	4.98	4.13	3.65	3.34

Time to Analyse: F test p-value

To get a p-value we compare our *F* statistic to an F(2, 42) distribution.



In our example

$$F = \frac{141.492/2}{33.3802/42} = 89.015$$

The p-value is really

$$P\left(F(2,42) \;>\; 89.015\right) = 0.0000000000\,0008$$

ANOVA Table

Results are often displayed using an ANOVA Table

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	141.492	2	40.746	89.015	p<0.01
Within Groups	33.380	42	.795		
Total	174.872	44			

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Within Groups	33.380	42	.795		
Total	174.872	44			

Pop Quiz!: Where are the following quantities presented in this table?

Sum of Squares Between (*SSB*)

Mean Square Error (*MSE*)

F Statistic

p value

ANOVA Table

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Sum of Squares Between (*SSB*)

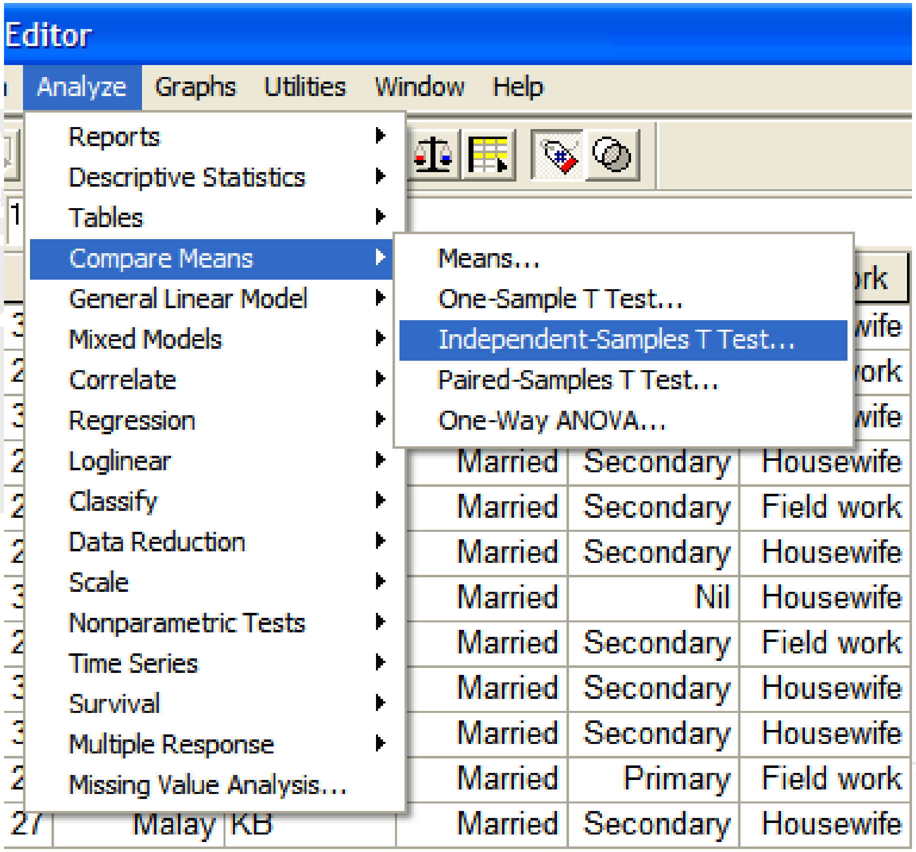
Mean Square Error (*MSE*)

F Statistic

p value

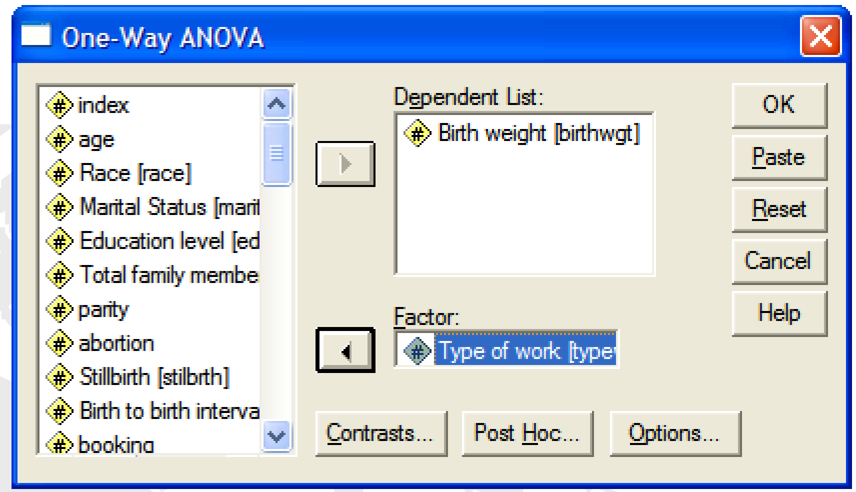
ANOVA In SPSS

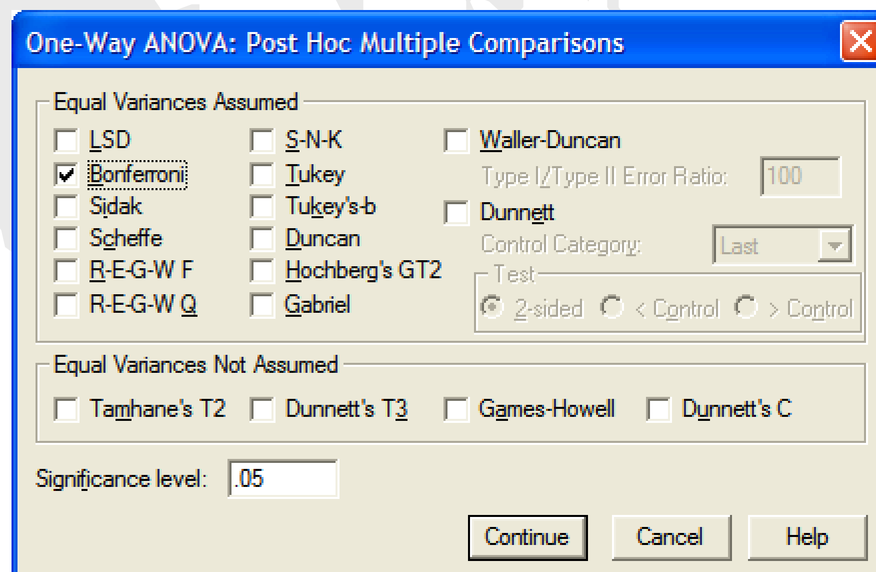
- For this exercise, we will be using the data from the CD, under Chapter 7, sga-bab7.sav
- This data came from a case-control study on factors affecting SGA in Kelantan.
- Open the data & select ->Analyse
  >Compare Means
    >One-Way ANOVA…

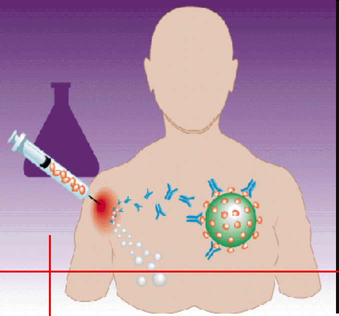


ANOVA in SPSS

- We want to see whether there is any association between the babies' weight and mothers' type of work. So select the risk factor (typework) into 'Factor' & the outcome (birthwgt) into 'Dependent'.
- Now click on the 'Post Hoc' button. Select Bonferonni.
- Click the 'Continue' button & then click the 'OK' button.
- Then click on the 'Options' button.

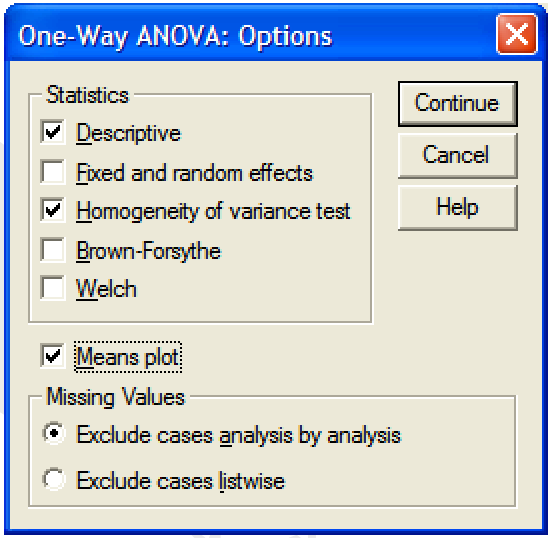


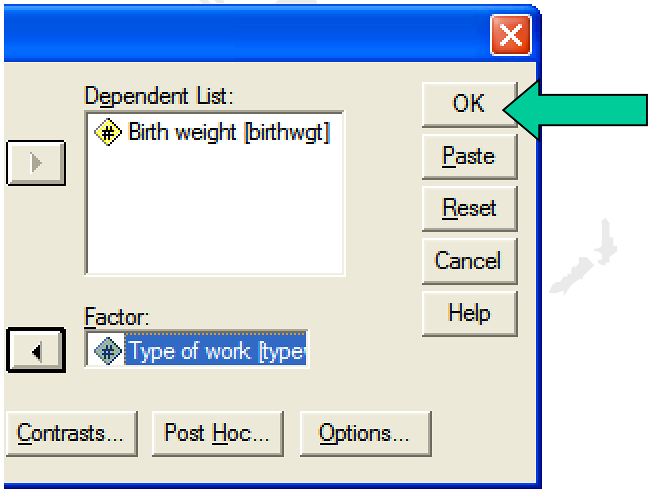


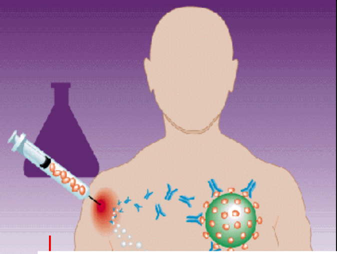


ANOVA in SPSS

- Select ‘Descriptive’, ‘Homegeneity of variance test’ and ‘Means plot’.
- Click ‘Continue’ and then ‘OK’.







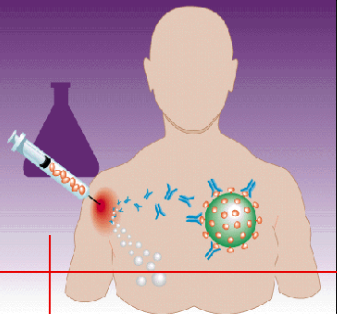
ANOVA Results

Descriptives

Birth weight

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Housewife	151	2.7801	.52623	.04282	2.6955	2.8647	1.90	4.72
Office work	23	2.7643	.60319	.12577	2.5035	3.0252	1.60	3.96
Field work	44	2.8430	.55001	.08292	2.6757	3.0102	1.90	3.79
Total	218	2.7911	.53754	.03641	2.7193	2.8629	1.60	4.72

- Compare the mean±sd of all groups.
- Apparently there are not much difference of babies' weight between the groups.



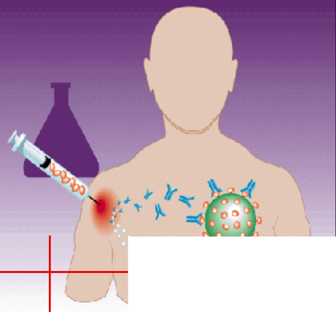
Results & Homogeneity of Variances

Test of Homogeneity of Variances

Birth weight

Levene Statistic	df1	df2	Sig.
.757	2	215	.470

- Look at the p value of Levene’s Test. If p is not significant then equal variances is assumed.



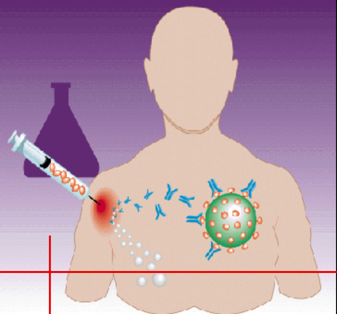
ANOVA Results

ANOVA

Birth weight

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.153	2	.077	.263	.769
Within Groups	62.550	215	.291		
Total	62.703	217			

- So the F value here is 0.263 and p =0.769. The difference is not significant. Therefore there is no association between the babies' weight and mothers' type of work.



How to present the result?

Type of Work	Mean±sd	Test	p
Office	2.76 ± 0.60	ANOVA F = 0.263	0.769
Housewife	2.78 ± 0.53		
Farmer	2.84 ± 0.55		